Math 256B. Homework 1

Due Wednesday, 31 January

Important: Please see the course web page

`https://math.berkeley.edu/~vojta/256b.html`

for important information on groundrules for homework assignments (including what "(NC)" means).

1. Do Hartshorne II Ex. 1.20.

2(NC). Carefully prove the following lemma (from class on Wednesday, 24 January):

Lemma. *Let $\mathscr{A}$ and $\mathscr{B}$ be abelian categories, let*

$$0 \longrightarrow I' \longrightarrow I \longrightarrow I'' \longrightarrow 0$$

be an exact sequence in $\mathscr{A}$, where I', I, and I'' are injective, and let $F\colon \mathscr{A} \to \mathscr{B}$ be a left exact, covariant functor.

Then the sequence

$$0 \longrightarrow F(I') \longrightarrow F(I) \longrightarrow F(I'') \longrightarrow 0$$

is also exact

In proving this, you may use (without proof) properties of abelian categories mentioned in Hartshorne's definition of abelian category, my definition of abelian category from class (originally from Wikipedia, but with additional information), or any other property of abelian categories mentioned in class *prior to* the time when the above lemma was stated in class. However, *do not* use Freyd's Embedding Theorem or any result depending on that theorem.

3. Let X be a noetherian topological space, and let $\mathscr{F}$ be a subsheaf of the constant sheaf $\mathbb{Z}$ on X. Show that $\mathscr{F}$ is finitely generated; i.e., that there are open subsets $U_1, \dots, U_n$ of X and sections $s_i \in \mathscr{F}(U_i)$ for all i such that no proper subsheaf of $\mathscr{F}$ contains all of these sections. [**Hint:** Consider the sets $\{P \in X : i \in \mathscr{F}_P\}$ for $i \in \mathbb{Z}$.]