Math 256B. Homework 12

Due Wednesday 24 April

1. State an analogue of Theorem 7.1 for $n = 0$. Prove it directly, using as few results from Section 7 as possible. (You may use the two definitions of dualizing sheaf and the fact that they are equivalent.)

2(NC). Let $\mathfrak{U} = (U_i)_{i\in I}$ be an open covering of a topological space X, and let I' be a subset of I such that $\mathfrak{U}' := (U_i)_{i\in I'}$ also covers X. Construct natural maps

$$\mathscr{C}^p(\mathfrak{U}, \mathscr{F}) \to \mathscr{C}^p(\mathfrak{U}', \mathscr{F})$$

for all $p \in \mathbb{N}$ and all sheaves $\mathscr{F}$ of abelian groups on X, functorially in $\mathscr{F}$, such that

(i). $\mathscr{C}^{\cdot}(\mathfrak{U}, \mathscr{F}) \to \mathscr{C}^{\cdot}(\mathfrak{U}', \mathscr{F})$ is a map of complexes for all $\mathscr{F}$, and

(ii). if X is a scheme and if $\mathscr{F}$, $\mathfrak{U}$, and $\mathfrak{U}'$ satisfy the hypotheses of (III, Thm. 4.5), then the maps $\check{H}^p(\mathfrak{U}, \mathscr{F}) \to \check{H}^p(\mathfrak{U}', \mathscr{F})$ induced by this map of complexes are compatible with the maps of (III, Thm. 4.5).

3. Let k be a field. Compute the dualizing sheaf of the (reduced) scheme

$$X = V(z) \cup V(x, y) \subseteq \mathbb{P}^2_k .$$

(This is the disjoint union of a line and a point.)