Math 256B. Homework 2

Due Wednesday 7 February

1. Do Hartshorne II Ex. 2.17.

2. Do Hartshorne III Ex. 2.2. (Note that the first sentence of (II Ex. 1.21) says that we're working with varieties over an algebraically closed field k, as defined in Chapter I, so $X = \mathbb{P}^1$ has no generic point.)

3. (Vakil 23.2.J,K)

   (a). Let Q be an injective abelian group, and let A be a ring. Show that $\operatorname{Hom}_{\mathbb{Z}}(A, Q)$ is an injective A-module. **Hint:** First describe the A-module structure on $\operatorname{Hom}_{\mathbb{Z}}(A, Q)$. You will only use the fact that $\mathbb{Z}$ is a ring, and that A is an algebra over that ring.

   (b). Show that $\mathfrak{Mod}(A)$ has enough injectives. **Hint:** Let M be an A-module. Find an inclusion $M \hookrightarrow Q$ of abelian groups, such that Q is an injective abelian group. Describe a sequence

$$M \hookrightarrow \operatorname{Hom}_{\mathbb{Z}}(A, M) \hookrightarrow \operatorname{Hom}_{\mathbb{Z}}(A, Q)$$

   of inclusions of A-modules. (The A-module structure on $\operatorname{Hom}_{\mathbb{Z}}(A, M)$ is via the action of A on the left argument A, not on the right argument M.)